A case for mixer thrust

Lars Uby, ITT Flygt, P. O. Box 1309, SE- 171 25 SOLNA, SWEDEN, lars.uby@flygt.com
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Abstract

Submersible mixers used mainly for flow controlled mixing in large volumes, are most commonly used in the jet mixing mode. As such, the single performance parameter *thrust (N)* is commonly known to be the basis for mixing system design, along with a wide set of mixer positioning principles. The motivation to use thrust as a sizing parameter is reviewed by considering the dynamic and kinematic properties of jets. Relations between thrust and primary flow rate and other parameters are discussed, to make a case for a more widespread use of the thrust parameter, i.e. in the context of mechanical agitation. This is also supported by the relative ease with which thrust can be measured, as indicated herein.

1. Introduction

Chemical/biological reactors rarely contain liquid bodies of the order of 10^3 m^3 or more, a most notable exception being wastewater treatment (WWT) basins. The need for mixing in these reactors, and in other liquid bodies of similar orders of magnitude, can be summarised in the following flow controlled mixing duties (in a wide sense of the word):

- Solids suspension
  - Off bottom suspension
  - Homogeneous suspension
- Blending
  - Continuous systems – prevention of short-circuiting
  - Batch systems
- Circulation
  - In certain WWT applications
  - Slow aeration
  - Prevention of physical, chemical, or biological processes (ice-freekeeping, stratification, eutrophication/algae growth)
  - Liquid transportation in open systems (open pumping)

Efficient flow controlled mixing is achieved by submerged jets. The larger the volumes, the more advantageous is the use of jets as compared to other mixing principles. However, even at relatively small volumes, where the differences between mechanical agitation and jet mixing are reduced, can there be advantages in jet mixing.

Although historically, jet mixing has been deemed to be energetically inefficient, the use of modern fully submerged jet mixers has changed that. Since the 1970's these mixers have been increasingly used in many areas where large liquid bodies need mixing. The span of volumes mixed by submersible jet mixers is indicated in Fig. 1.

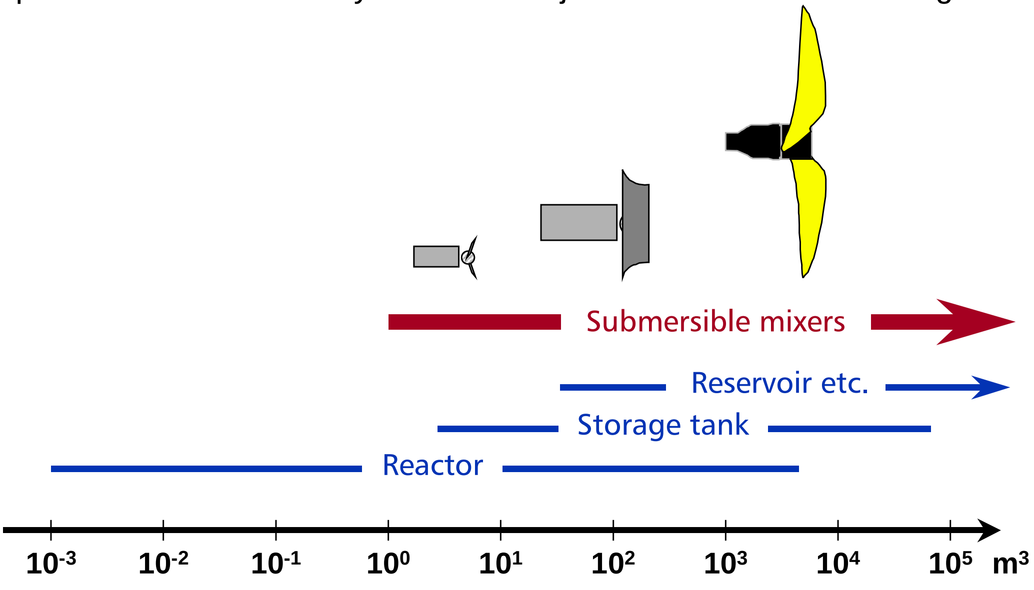

Fig. 1. Volumes typically mixed by one or more fully submersible jet mixers.

In reactors where mixing is critical to successful long term process results, mixing system design is most of the time acknowledged by the industry to be of prime importance. However, in WWT where the process result rather should fulfill legal directives, than produce economical profit, little attention has been paid to such design. As WWT is the largest market for submersible mixers, suppliers have been much left to themselves to develop the system design criteria, with little interest from industry or academia. WWT is a globally growing activity with strongly environmental bias, whence an increased interest in wastewater reactor engineering has been noticed.

It has therefore been deemed an appropriate point in time to collect the experience of the leading submersible mixer suppliers and to produce standardised ways of characterising mixers, as a first step toward a better understanding of the mixing concept among the main users.

The two main characteristics of a submersible mixer are its *thrust (N)* being its output, and its *electric power consumption (W or kW)* being the main part of the Life Cycle Cost. In the following sections, motivation for using the thrust parameter is discussed, and its relation to other parameters is given. The physical reasons for considering thrust or momentum flux in hydrodynamic problems, such as mixing system design, are only briefly mentioned, but should not be difficult to grasp. The simplicity of the thrust (or thrust number) measurement procedure, as opposed to e.g. flow number measurement, is also suggested. The organisation of the ongoing standardisation is briefly mentioned in the end.

2. Turbulent jet properties

Since the submersible mixers usually operate in a jet mixing mode, it is appropriate to first recapitulate some basic facts about submerged jets. For simplicity, turbulent round jets away from fluid boundaries will be considered.

A submerged jet, as depicted in Fig. 2, issues from a pipe/nozzle or is generated by an axial flow impeller (typically a propeller). The majority of literature data on swirling and non-swirling jets refer to nozzle jets, whereas most gross features to be considered here are not expected to be different for impeller jets. Whether the impeller is shrouded or not, there is a conceptual difficulty in defining the initial jet cross section area A_0. The common approach is just to set it equal to the circular area swept by the impeller – at the *impeller disc*, or delimited by the shroud if present.

During a pressure equilibration phase, the issuing jet accelerates and narrows into the *vena contracta.* The minimum cross section area of the vena contracta is $A_0 / 2$. The assumption of e.g. flat axial velocity profile is more adequate at this cross section than at any other. Quantitatively, this is to say that the inequality $J \geq \rho\, Q^2 / A$ is closer to equality here than elsewhere.

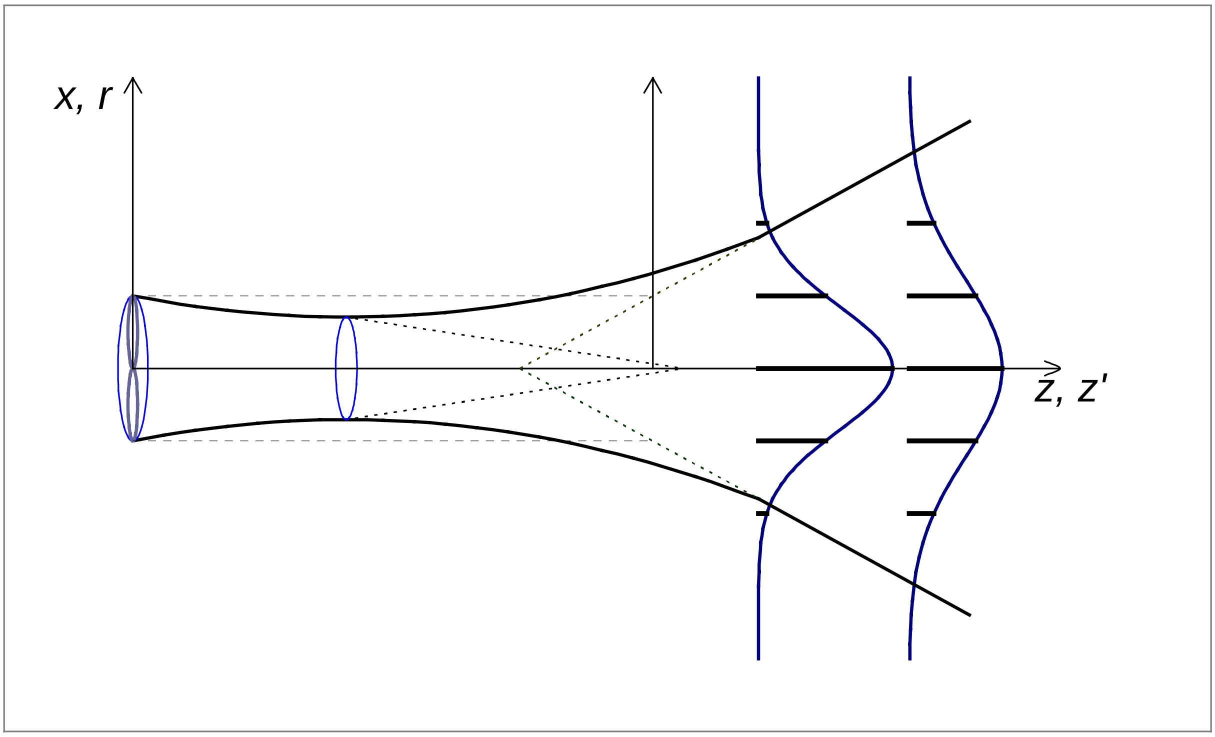

Fig. 2. Definition sketch showing jet initial cross section at the propeller, vena contracta cross section where transverse pressure gradients vanish, potential core tapering off, virtual origin for selfsimilar jet, origin for the coordinate *z'*, and selfsimilar axial velocity profiles.

Then entrainment of ambient fluid reduces the *potential core* of the jet up to the point where the velocity profiles are completely determined by the entrained fluid. Here, velocity profiles are self-similar. Hanel (1977) briefly describes early work considering the dependency of profiles on initial details of the jet. In more recent work the influence of boundary conditions has been revisited (George 1989, Hussein & al 1994, Mi & Al 2001). It seems that a scaling based on turbulence intensity can be used to collapse again the velocity profiles (Papadopoulos & Pitts 1999). In the following, the profiles will mainly be used to determine mass, momentum and energy fluxes in a submerged jet, whence the standard profiles will be used without qualification. Using the coordinate system in Fig. 2, the profiles may be written (velocity $\boldsymbol{u} = U\boldsymbol{e}_z + V\boldsymbol{e}_r + W\boldsymbol{e}_\varphi$)

$$U = \frac{s_0}{z'-a} e^{-C_U \xi^2} \tag{1}$$

$$V = \frac{s_0}{z'-a} \xi \left[e^{-C_U \xi^2} \left(\xi^2 + \frac{1}{2C_U} \right) - \frac{1}{2C_U} \right] \tag{2}$$

$$W = \frac{q_0}{(z'-a)^2} \xi e^{-C_W \xi^2} \tag{3}$$

Velocities integrated over the jet cross section at some value of the coordinate *z* will be integrated up to infinite radius, rather than the jet radius $r_\varepsilon(z)$ where the axial velocity is ε times the velocity on the axis. In many cases, it can be seen that there is only a small error involved in integrating to infinity. For instance, the expressions for Q, J and P_z in Eqs. (4) – (6) will be wrong by *5%, 0.25%* and *0.0125%* respectively as compared to a calculation for $\varepsilon = 0.05$. The volumetric flow rate Q, the asymptotic linear momentum flux J[1], axial power P_z, angular momentum flux M and swirl S are given by

$$Q(z) = \frac{\pi s_0}{C_U}(z'-a) \tag{4}$$

$$J = \frac{\rho \pi s_0^2}{2C_U} = \rho Q_0^2 \frac{C_U}{2\pi a^2} \tag{5}$$

$$P_z(z) = \frac{\rho \pi q_0}{6C_U (z'-a)} = \frac{2J^2}{3\rho Q(z)} \tag{6}$$

$$M = \frac{\rho \pi s_0 q_0}{2(C_U + C_W)^2} \tag{7}$$

$$S = \frac{2M}{JD} = \frac{2q_0}{Ds_0} \frac{C_U}{(C_U + C_W)^2} \tag{8}$$

Negative contributions to the momentum flux from static and fluctuating dynamic pressure have been omitted, as they are assumed to approximately balance except very near the impeller, $z << D$. Velocity measurements at a plane $z << D$ will not exactly capture the asymptotic momentum flux because of the pressure. Difficulties with this assumption have been encountered in closed (recirculation) systems, but the more complex picture seems to depend on the difficulties in measuring properties of a confined and even sideways oscillating jet (Hussein & Al 1994, Peterson & Al 2000). For the swirling jet, $W \neq 0$, a pressure contribution expressible in W^2 is notable but quickly reduced along the jet.

The axial power was defined for the axial mean flow contribution only. Most notably, axial power decreases along the jet and flow rate increases. The axial fluxes of kinetic energy corresponding to azimuthal and radial flow are

$$P_\varphi = \frac{\pi \rho s_0 q_0^2}{2(C_U + 2C_W)^2} \frac{1}{(z'-a)^3} \tag{9}$$

$$P_r = \frac{\pi \rho s_0^3}{C_U^2} \left(\frac{1}{9} + \frac{1}{8} \ln \frac{4}{3} \right) \frac{1}{z'-a}, \tag{10}$$

and are seen to vanish rapidly. The flow entrainment is

[1] By momentum flux is meant that associated with net mass flux. This is what is detectable by velocity measurements alone, disregarding any pressure gradient.

$$K_e = \frac{D}{Q_0}\frac{dQ}{dz} = \frac{D}{a}\,. \qquad (11)$$

The linear and angular momentum fluxes are conserved along a free jet unless body forces oppose it, exactly so if pressure terma are included in the fluxes. Insofar as the jet generator accelerates stillstanding fluid, they are equal to the mixer thrust and torque respectively. More precisely, the thrust F imparted to a fluid by a propeller and the momentum flux J can be expressed in the following way:

$$F = \int \rho\, U\, \Delta U\, dA \qquad (12)$$

$$J = \int \rho\, U^2\, dA \qquad (13)$$

where ΔU is the axial velocity increase generated over the action range of the propeller, i.e. longer than $z << D$. The value of J, excluding pressure gradient, *increases* along the jet to its asymptotic value. In what follows, no substantial difference will be made between the magnitudes of the thrust and the momentum flux. In an impeller mixing application, some difference will be present, but it can be handled in various ways.

The angular momentum flux M is related to torque in the same way that linear momentum is related to thrust. However, the significance of torque seems to be undisputed in mechanical agitation, or at least implicitly accepted via the use of power number. It will not be further discussed here.

3. Performance parameters and dimensionless numbers

The two most commonly used characteristics of an axial flow type mechanical agitator are the flow and power numbers. Their definitions are based on turbulent flow conditions, and so they vary strongly for low Reynolds numbers. The present discussion is limited to the fully turbulent regime, unless the opposite is explicitly stated. To put the thrust number in its context, the definitions of all three parameters are given:

$$Fl = \frac{Q_0}{ND^3} \qquad (14)$$

$$Th = \frac{F}{\rho N^2 D^4} \qquad (15)$$

$$Po = \frac{P_0}{\rho N^3 D^5} \qquad (16)$$

Note that a momentum number *Mo* can be defined analogously to the thrust number:

$$Mo = \frac{J}{\rho N^2 D^4}\,, \qquad (15')$$

to be compared with Eqs. (32) and (33). The relations between the three numbers are discussed in Secs. 3.1 and 3.2.

An efficiency number for an impeller in a certain flow situation relates output to input. The common relation used for non-dimensional efficiencies is that of output power to input power, discussed in Sec. 3.3. For output power, the axial flow power P_z will be used. This simplificaton is motivated by the fact that axial flow impellers and jet mixers typically act in truly flow-controlled mixing processes. That is, other flow components than the mean axial flow, such as the azimuthal/ tangential, are abundant relative to the needs.

There is motivation to consider a combined mixer-system efficiency. The reduction of this efficiency to the dimensional *thrust-to-power ratio* parameter based only on the mixer is described in Sec. 3.4.

In Sec. 3.5, it is shown how the mixer based Reynolds number can be defined in various ways depending on what aspects of flow are to be characterised, or what comparisons are to be made.

Eventually, independent work on application of the parameter discharge momentum flux *J*, often just called momentum, will be mentioned in Sec. 3.6.

3.1 Flow and thrust numbers

Whereas *Fl* and *Th* are directly related to the axial motion generated by the mixer, the significance of *Po* in terms of fluid kinematics is more involved. The relation between *Fl* and *Th* can proposedly be written

$$Fl = c\ Th^{1/2}. \tag{17}$$

The value of *c* can be estimated in various ways:

3.1.1 Direct measurement of thrust and indirect flow rate measurement on a range of Flygt submersible mixers have resulted in a value $c = 0.84 \pm 0.04$.

3.1.2 Where flow, power and swirl numbers are known, *c* can again be inferred. For instance, Bakker and Van den Akker (1994) report the following data:

Impeller	Fl	Po	S
A315	0.74	0.76	0.31
PBT	0.81	1.55	0.52

From the definition of *S,* provided in Eq. (8),

$$c = Fl\left(\frac{\pi S}{Po}\right)^{\frac{1}{2}} \tag{18}$$

and $c = 0.84$ is obtained for the A315, and $c = 0.83$ for the PBT.

3.1.3 Given Eqs. (4), (5) and (11) for the self-similar jet

$$c = \frac{1}{K_e}\left(\frac{2\pi}{C_U}\right)^{\frac{1}{2}} \quad (19)$$

The data of Chigier and Chervinski (1967) indicate that for $S < 0.25$, $0.80 < c < 0.83$. For moderate to strong swirl ($S \geq 0.4$) the value of c would be smaller, though this has not been confirmed. As the abovementioned dependence of jet properties on initial data has not been acknowledged until recently, reports on different values of entrainment rate, velocity decay, position of virtual origin, etc., have seemed to be contradictory. Therefore, the full picture has not yet been collected.

3.1.4 Assuming a linear velocity profile ($U \propto r$, $r \leq D/2$) as often done for quick estimates on propeller jet initial properties, $c = 0.8355 \approx 0.84$ results. For a flat profile, $c = 0.886$, which is the maximum possible value.

These four examples indicate that although in general the value of c depends on the velocity profile, profiles are typically such that the same value occurs. Hence the translation between thrust and flow rate can be expected to be rather painless for axial flow impeller mixers. This conclusion is mainly interesting when previous mixing correlations based on flow number are translated into a thrust number formulation. Where a value of c is required in this paper, $c = 0.84$ will be used.

As indicated in Sec. 4, it is also probable that mixer vendors can provide thrust number data for their own products. However, not always are impellers purchased from specialist companies. Some of them, such as a PBT, can also with ease be produced in-house by e.g. a chemical company.

3.2 Flow and thrust numbers vs. power number

In studies of impeller efficiency (cf. Sec. 3.4 below) the idea of a correlation between *Fl* and *Po* was put forward (Herbert & Al 1994, Ruszkowski 1992). The correlation is of the form $Fl = \alpha Po^{\beta}$, where it seems from various data that $0.70 < \alpha < 0.80$, and $\beta \leq 1/3$. However, as the energy efficiency $\propto Fl^3 / Po$ of a commercially useful impeller probably falls between 30% and 90%, the correlation is probably more of a rule of thumb signifying this, cf. Sec 3.4 below. It is not a meaningful relation with a deeper meaning. Probably the exaggerated use of log-log diagrams over this very limited numerical range has strengthened the impression of true correlation.

Similarly, Grenville & Al (1999) discussed the relationship between *Mo* and *Po.* As From the above argument and the findings in Sec. 3.1, it is reasonable that similar correlations be found. Indeed, Grenville found $Mo = \gamma Po^{\delta}$, where $0.70 < \gamma < 0.80$, and $0.60 < \delta < 0.70$.[8] In addition, $\gamma = (\alpha / c)^2$ and $\delta = 2\beta$ are approximately contained in these data, as expected provided one can identify *Mo* with *Th.* It should be noted that Grenville & Al defined two momentum numbers (cf. Sec 3.6), corresponding to mean and fluctuating axial flow respectively. As indicated in Sec. 2 above, only the mean flow contribution was considered here. (That is to say, it is assumed here that static pressure counteracts the fluctuating part.) Still, the interpretation of this correlation is not much different from that of the previous one.

The thrust and torque (hence power) number contributions of an impeller blade element are proportional to

$$C_L \cos\varphi - C_D \sin\varphi \tag{20}$$

$$C_L \sin\varphi + C_D \cos\varphi \tag{21}$$

respectively, where φ is the angle between (the tangential) blade element velocity and the incident fluid velocity. The acuteness of the angle φ indicates a close relationship between thrust and impeller blade lift characteristics on one hand, and between power and impeller blade drag characteristics on the other hand. This is not an absolute division – the blade lift can contribute considerably to the torque, but it serves to explain that there need not be a more fundamental simple relation between thrust and power, or between axial flow and power.

3.3 Efficiency

The output to input power ratio is, according to the abovementioned selection of P_z as the relevant output,

$$E = P_z / P. \tag{22}$$

Using Eq. (6) and subsequently Eq. (15), the following is found:

$$E = 2\,F^2 / 3\,\rho\,Q\,P = 2\,Th^2 / 3\,Fl\,Po = \tag{23}$$

$$= (\,2 / 3\,c\,\rho^{1/2}\,)\,F^{3/2} / D\,P = (\,2 / 3\,c\,)\,Th^{3/2} / Po = \tag{24}$$

$$= (\,2 / 3\,c^4\,)\,Fl^3 / Po \tag{25}$$

In SI units, the numerical prefactor in the LHS of Eq. (24) is ≈ 1/40 for water. Eq. (25) may be compared with e.g. the flat velocity profile efficiency, generally known to be $(\,8 / \pi^2\,)\,Fl^3 / Po$. The numerical prefactor in this expression is 60% of that in Eq. (25). The flat profile efficiency could also be expressed as $(\,8\,c^3 / \pi^2\,)\,Th^{3/2} / Po$ using Eq. (17). In Fig. 3 a set of data collected by Fentiman & Al. has been plotted using Eq. (25).

It is also interesting to consider the power as expressed by pQ. Given that the profiles are at their flattest at the waist of the vena contracta, where the cross section area is half the initial jet area, flow rate can be calculated there:

$$p = F / A_0 \tag{26}$$

$$Q = (\,F\,(A_0 / 2) / \rho\,)^{1/2} \tag{27}$$

lead to

$$p\,Q / P = (\,2 / \rho\,\pi)^{1/2}\,F^{3/2} / D\,P. \tag{28}$$

The numerical prefactor is again ≈ 1/40 for water, as in Eq. (24). However, much flow and pressure field detail is conceiled in this coincidence.

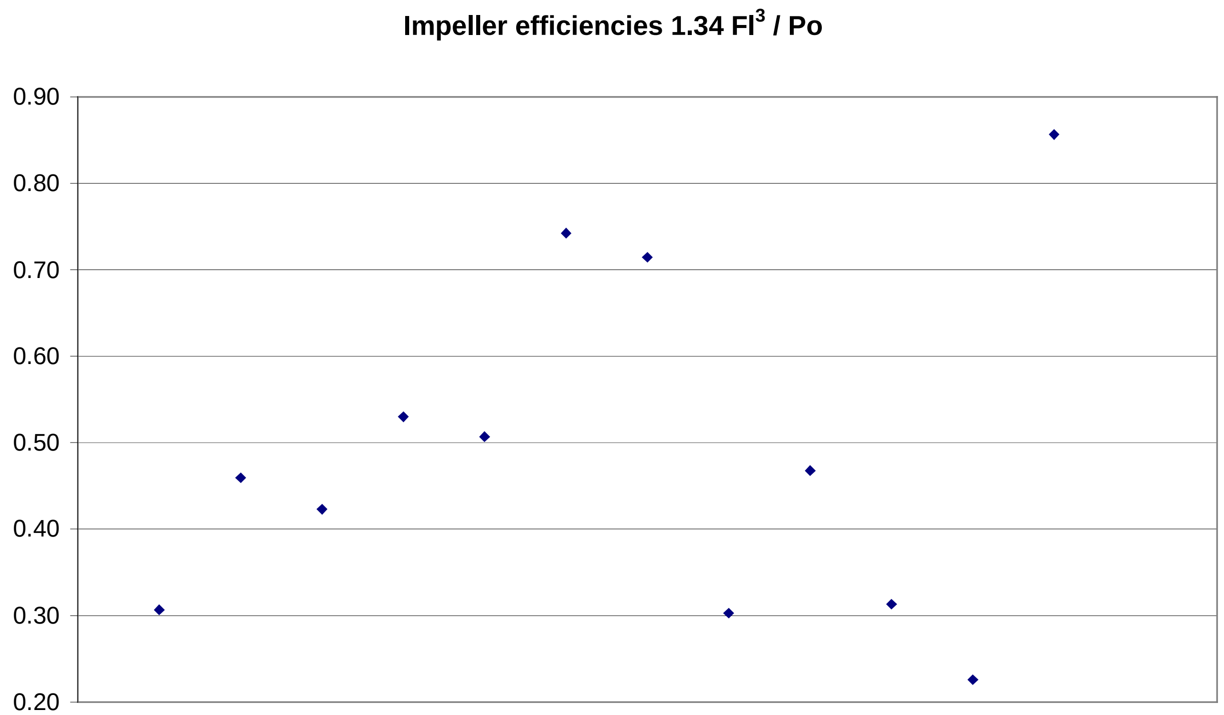

Fig. 3. Impeller data given in Fentiman & Al. plotted over main part of the admissible range. The data is scattered throughout the admissible range (*0* to *1*), and is not confined to a narrow band that would indicate a strong cubic correlation between *Fl* and *Po.* The efficiency is that given by Eq. (25) and $c = 0.84$.

3.4 Thrust-to-power ratio

There is a more systems-oriented way of considering the impeller efficiency. Consider the thrust F required of an impeller to maintain a state of motion in body of liquid. It equals the losses of absolute momentum per unit time, and hence the power requirement is Fu, u being a typical bulk flow velocity of the system. (In fact, as submersible mixers often operate in circulation channels, this view may be adopted even without provisions for complexity added by flow geometry.) The power of the impeller, on the other hand, is $F(u + \Delta U/2)$, where the velocity increase is interpreted as an averaged quantity, and the geometry is assumed to allow for using the same system velocity u. Hence, the system-to-impeller power ratio is

$$E_{SI} = (F / P)\, u = 1 / (1 + \Delta U / 2u). \qquad (29)$$

This illustrates the fact that slower/larger impellers, producing a smaller acceleration across the impeller disc, are more efficient flow generators. Considering a mixer/impeller per se, the thrust-to power ratio

$$R \equiv F / P = 1 / (u + \Delta U / 2) \qquad (30)$$

(often thought of with $u = 0$) comes close to the impeller-system efficiency. Note that $R \propto 1 / ND$. For submersible mixers, this ratio usually falls between 0.1 and 1 N/W, when the electric power is considered.

3.5 Reynolds numbers

The most common form of the impeller Reynolds number is closely related to the thrust. In fact, it is sometimes convenient to define a modified Reynolds number

$$\mathrm{Re}_{Th} = Th^{1/2}\,\mathrm{Re} = \frac{(\rho F)^{1/2}}{\eta}\,. \qquad (31)$$

This comes closer in behaviour to the corresponding Reynolds number for pipe/nozzle jets. For a more distinct comparison, the factor $4c/\pi = 1.07$ is also required on Re_{Th}. This Reynolds number also makes meaningful a comparison of systems with geometrically similar but not exactly equal impellers.

Both F and Re (or Re_{Th}) are conserved along an unbounded jet. Discussions on whether the momentum flux or the jet Reynolds number is the best system design parameter should be put in a perspective of what forces are important for the present problem. Whereas F carries information on mixing driver inertia only, Re carries information on its relation to viscosity. This relation may or may not influence the mixing result.

One could envisage a solids suspension system where viscosity may increase by some factor, to still leave the jet fully turbulent, but to render solids settling velocity much smaller. The “system viscosity” has influenced the mixing result, but the viscosity felt by the jet in particular has not. This speaks for F as a more significant parameter, although the changes in requirement on F and on Re could be misinterpreted to be equally fundamental to the application. (The philosophy of separation of “mixer” or “mixing driver” on one hand, and “system” on the other, frequently helps in avoiding such pitfalls. It should be kept in mind that physically the separation may be very blurry, but theoretically it is still useful. It is at its best in jet mixed applications, and needs to be applied with much care in mechanical agitation.) Lane and Rice (1982) have touched on the difference between the jet and the tank recirculation flow in the blending context.

Re_{Th} could contain more detailed information on the influence of liquid properties, since the thrust number is dependent on flow regime. Unfortunately this would make the practical definition of Re_{Th} more complicated, and therefore the value of Th in Eq. (31) is often taken to be that of fully turbulent flow.

3.6 Discharge momentum in mixing literature and practice

In jet mixing, jet Reynolds number and momentum flux – the word “flux” is often omitted in recent literature – have long been used for system design. Reference is often made to Fossett and Prosser (1949), who appreciated momentum flux conservation, or Fox and Gex (1956), who made more systematic use of the momentum concept. In fact, Fox and Gex described their results in terms of Reynolds number and momentum flux for blending by both jet mixing and mechanical agitation. Although they did not collapse the data of the two mixing methods, it is apparent that this can be done for the turbulent regime without effort. Probably, $(4c/\pi)^{5/6}Th^{5/12}$ is the factor relating the two, but this cannot be confirmed without knowledge of the propeller they used. For the transitional regime, a slightly stronger dependence on viscosity was found for jet mixing than for mechanical agitation. The relation of energy consumption to discharge source size, given equal mixing result, was clearly stated (cf. Sec 3.4).

Several studies have been performed to confirm or reject these early findings. Grenville and Tilton (1996) have made a survey and analysis of several sets of jet blending data. Studies by e.g. Lane and Rice (1982) and by Simon and Fonade (1993) have further discussed the relation of jet Reynolds number and jet momentum to blending time. Unfortunately, the variety of geometries and assessment methods may have caused data sets to be less comparable, although Fox and Gex (1956) claim that blend time is independent of geometry – only the position of the last mixed region was found to depend on it.

It was soon argued that the pumping capacity would better (than power based parameters) decribe stirred tank kinematics, which in its turn would allow more detailed understanding of the mixing process. After Cooper and Wolf (1968) eventually measured primary flow rate, the flow number was established as one of the most important parameters. Momentum flux seems to have been ruled out by most, but not all, since those days.

Recently, Grenville &Al (1999) have considered jet momentum and impeller momentum correlations with blend time, using experimental data from later studies. Two momentum numbers were defined,

$$Mo = \int_{Ao} U^2\, dA \,/\, N^2 D^4 \qquad (32)$$

$$mo = \int_{Ao} u'^2\, dA \,/\, N^2 D^4 \qquad (33)$$

corresponding to the mean and fluctuating velocity contributions respectively. (In the current paper it is assumed that the fluctuating part is approximately balanced by pressure terms, cf. Sec. 2.) As the momentum numbers were known, it was easier to collapse blend time data for the two mixing methods. However, only turbulent mixing was considered.

ITT Flygt have used the thrust parameter internally since1978, and colleagues in submersible jet mixing have followed, cf. Sec. 5. Apart from industrial jet mixing, it should also be noted that momentum has essentially been used for many years in mechanical agitation by Prochem, under the name *“QV”*. Similarly, the SCABA number *“N_{SC}”* used for scaling of equal geometry impellers is practically proportional to the 5/4th power of the thrust.

4. Measurement methods for mixer thrust

As mentioned in Sec. 2, there are two closely related parameters, thrust and momentum flux, usually just named momentum. In the case of a nozzle jet, the distinction between the two is irrelevant. For the impeller generated jet, they will coincide in a situation where all momentum of the jet flow is directly contributed by the impeller.

Considering the quasisteady state of a mixing flow driven by an impeller in a vessel, i.e. disregarding the startup phase, the flow field is easier to predict on the basis of the impeller thrust than on the total jet momentum. The flow field is that which can be upheld by the thrust given the momentum losses in the system. This is principally similar to a loss calculation for pumping liquid through a pipe. In contrast, the total jet momentum consisting of the impeller thrust part and of the momentum of the quasisteady flow is a more complete description of the jet characteristics – but not necessarily of the whole flow in the vessel. The loss calculation aspect is a motivation to consider thrust (or thrust number) measurement for impeller characterisation.

Historically, another motivation to prefer thrust to momentum flux was the relative simplicity of the thrust measurement. In addition, the thrust enters as a momentum source boundary condition in the hydrodynamic system, whereas the momentum flux as measured in a section at $z << D$ does not include pressure gradient influence on the flow field.

The thrust characteristics of an impeller is usually such, that in an ambient coflow the thrust is markedly reduced. This is easy to understand, for at high coflow speed the impeller will not be able to accelerate the liquid, and hence will not act on it with a force. Additionally, the angle of attack and the advance ratio at the impeller blades will be different. Pressure effects may also influence the performance. More generally, in any ambient flow the thrust will deviate from that in a quiescent environment. It is therefore important to define the conditions under which an impeller should be characterised.

The infinity of geometric and kinematic configurations arising in submersible mixer applications has implied that a single reference flow situation should be used – that of a quiescent ambient. This is usually referred to a *open sea conditions.* Performance in radically different flow environments has been measured, and can be inferred from basic data. An arrangement providing *virtual open sea conditions* is depicted in Fig. 4 (left). Since mechanically agitated vessels are designed on the basis of correlations for standardised geometries, use of these geometries would be a preferred direction to choose for thrust number measurements.

The measurement principle draws on Newton's third law of motion. In this context, it can be stated: *A mixer acting on a fluid with a force F is acted on by the fluid with a force –F.* This force can be measured using load cell arrangements, as for instance in Fig. 4. Hence, the measurement does not involve taking velocity readings over a cross section of the discharge, and the definition of such a cross section is therefore not needed. The measurements are highly reproducible, as has been confirmed by measurement at different sites. A device for in-situ thrust measurements is also depicted in Fig. 4 (right). It can be noted that in propulsion, similar measurements are commonplace, and marine technology centres have been engaged for measurement by some vendors of submersible mixers.

The importance of the impeller thrust for the mechanical design of a mixer is of course recognised by mixer vendors. For this reason, if not for any other, most vendors are probably already able to provide thrust data, if asked for. The measurement principle is probably similar to the one shown here.

It can be added here, that from a theoretical point of view, the generalised radial force or the radial momentum flux of a radial impeller, may also be a useful characterisation parameter. However, measurement of any of these entities does not seem to be any simpler than measurement of flow rate. This should of course not deter from theoretically approaching the idea.

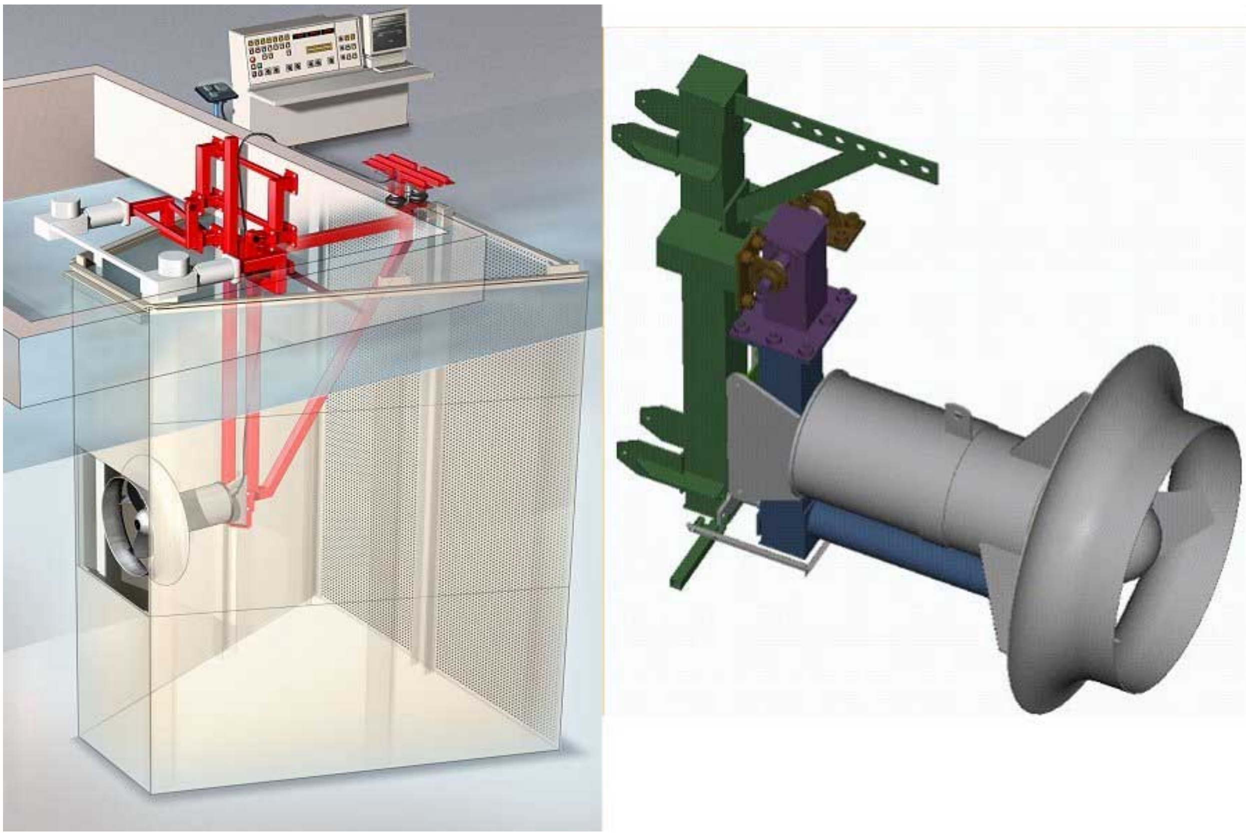

Fig.4. (Left) Mixer thrust measurement arrangement in use by ITT Flygt since 1993. The mixer rests on a bar hanging on the black-marked pivots. The reaction force of the mixer is transmitted to a load cell suspended between two pairs of bellows. The baffle arrangement and perforated backplate ensure that recirculation flow evanesces. After quasi-steady flow is established, the load cell signal is recorded. Tara readings are taken equally carefully. (Right) Mixer thrust measurement device for use in situ. The green bar is mounted on the mixer installation guide bar. A load cell is situated between the green bar and the vertical pivoted blue bar. A device for measuring both thrust and torque, containing 5 load cells – 3 for thrust and 2 for torque – suspended on a rectangular frame, is also in use at ITT Flygt.

5. Ongoing standardisation

In early 2000, a document prepared by a group of mixer vendors in Sweden and a representative of the Swedish Standards Institute (SIS), was submitted to the International Standards Organisation (ISO). The document concluded in a proposal to standardise measurement of mixer performance and mixing results. After voting, ISO decided in June 2001 to approve. Call from ISO to form working parties is expected.

Because of the uncertainty of this voting, already in early 2001, contact was made by ITT Flygt with other manufacturers of submersible mixers through the Europump organisation's Standards Committee. A working party was formed to formulate a mixer performance measurement standard for Europump members. The objectives were to ensure the production of a standard regardless of the vote at ISO, and to accelerate the work toward standard practice among Europump members.

This work has so far resulted in a document similar in spirit to the ISO 9906 standard for pump performance measurements. It has also been instrumental in sorting out differences of aspect and misconceptions among the vendors, and has confirmed that the branch is maturing toward a state where standards can be formulated. The document will be published and generally available when it has been completed in detail. This is expected during early 2002.[2]

6. Summary and conclusion

From the point of view of jet mixing, with either nozzle jets or impeller jets, the usefulness of the thrust (or thrust number) parameter for mixing system design has been elucidated. The relation between thrust and flow rate appears to be well known, and the simplicity of thrust measurement was indicated. It is argued that years of successful measurement and application of mixer thrust by ITT Flygt, followed by other manufacturers of submersible mixers, is a strong motivator for vendors, users and researchers to embrace the concept even for classical mechanical agitation. The practice is not unknown, but is only little spread.

7. Acknowledgement

During the presentation of this paper at the IChemE FMPSG meeting, professor Matthias Kraume, Institut für Verfahrenstechnik, TU Berlin, kindly pointed out some references that could serve as further reading. These are

- *Rührwerke. Theoretische Grundlagen, Auslegung und Bewertung (~ Agitators. Theoretical foundations, design and evaluation);* Liepe, F., Sperling, R. and Jembere, S. (Eigenverlag Fachhochschule Köthen, 1998), Chap. 2.3 and 3.3.3. This is an excellent introduction to mixer thrust, and elaborates especially on contents of Secs. 3.1- 3.4 of this paper.
- *Verfahrenstechnische Berechningsmethoden (~ Computational methods of process engineering);* Edited by Weiß, S. & Al. (VCH Verlagsgesellschaft, 1988), in particular Teil 4, 2.4.6 by Liepe, F.
- Fort, I., Coll. Czech. Chem. Com. **34,** 3673 (1969). The latter two provide experimental thrust number data, Fort being the authoritative source, referred to by the former.
- *Concept for scale-up of solids suspension in stirred tanks;* Kraume, M. and Zehner, P., submitted to Canadian Journal of Chemical Engineering. This paper shows the simplicity with which important results can be reached if the physical entity *force* is taken into consideration.

[2] Questions about the Europump Submersible Mixer Testing Code, SMTC, may be addressed to the author of this paper.

8. Notation

Symbol	**Unit**	**Description**
A	m^2	Jet cross section area
A_0	m^2	Initial cross section area of jet
C_D	-	Drag coefficient
C_L	-	Lift coefficient
C_U	-	Axial velocity profile parameter
C_W	-	Azimuthal velocity profile parameter
D	m	Propeller diameter or nozzle diameter
E	-	Impeller efficiency
E_{SI}	-	Combined system-impeller efficiency
F	N	Thrust
Fl	-	Flow number
J	N	Momentum flux
K_e	-	Entrainment constant of jet
M	Nm	Torque
Mo	-	Momentum number, total or corresponding to mean flow
P	W	Power
P_r	W	Power of radial motion
P_z	W	Power of axial motion
P_0	W	Initial power of axial motion of jet
P_φ	W	Power of azimuthal motion
Po	-	Power number
Q	m^3/s	Flow rate
Q_0	m^3/s	Initial jet flow rate (i.e. at impeller or nozzle)
R	N/W	Thrust-to-power ratio
Re	-	$= \rho ND^2 / \eta$, impeller Reynolds number
Re_{Th}	-	$= Th^{1/2} Re$, modified Reynolds number
S	-	Jet swirl
Th	-	Thrust number
U	m/s	Mean part (as opposed to fluctuating) of axial velocity
ΔU	m/s	Axial velocity increase generated by impeller
V	m/s	Azimuthal velocity
W	m/s	Radial velocity
a	m	Axial position (z') of virtual origin of jet
c	-	Constant relating thrust number to flow number
e	-	Unit vector
mo	-	Momentum number corresponding to fluctuating flow
p	Pa	Pressure
q_0	m^3/s	Azimuthal flow rate amplitude of jet
r	m	Radial coordinate of jet
s_0	m^2/s	Axial transport amplitude of jet
u'	m/s	Fluctuating part of axial velocity
u	m/s	Velocity vector
u	m/s	Integral velocity of mixed system
x	m	Cartesian coordinate transverse to jet
z	m	Axial coordinate

z'	m	Axial coordinate of jet, zero where selfsimilar jet area equals disc area
α	-	Flow number – power number relation factor
β	-	Flow number – power number relation exponent
γ	-	Thrust number – power number relation factor
δ	-	Thrust number – power number relation exponent
ε	-	Axial velocity amplitude relative to jet center axial velocity
η	kg/ms	Dynamic viscosity of fluid
ξ	-	= r/(z' - a), selfsimilar coordinate
ρ	kg/m^3	Fluid density
φ	-	Angle between impeller blade element velocity and incident fluid velocity

9. References

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