POWER DISSIPATION, THRUST FORCE AND AVERAGE SHEAR STRESS IN THE MIXING TANK WITH A FREE JET AGITATOR

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Power Dissipation, Thrust Force and Average Shear Stress in the Mixing Tank with a Free Jet Agitator

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The momentum transferred from a mixer jet to the liquid in the vessel produces mixing action and circulation within the tank. The jet from a submersible mixer is similar in kind as that from a nozzle and has freedom of location in the vessel. This is called Free Jet Flow Agitator (FJFA). This paper deals with the general concept of the FJFA sizing method, based on force (thrust) input from the propeller to the media.

One needs to understand the objective [3,6,9,14,17] of the fluid/solid mixing process as the first step in determining which type mixing equipment should be used. Although mixing can be accomplished by many different types of mechanisms and equipment, there are only a few types which produce effective mixing results for any individual process. We all know that there are different energy dissipation requirements for each different process. The energy dissipation characteristics for non-submersible mixers (agitators) has been covered in various books, technical-society magazines and symposium papers.

This presentation covers the use of submersible mixers (agitators) which produce a high velocity jet of liquid within a tank. The momentum transfer from the mixer jet to the liquid in the vessel causes the mixing action [6,15] and circulation within the tank. Although the submersible mixer jet is comparable to the jet from a nozzle, it holds an advantage of freedom of location in a vessel. The submersible-mixer type equipments are called Free Jet Flow Agitators (FJFA).

There has not been much information published on FJFA/mixer sizing methods. According to Tatterson [14] jet mixing is very often overlooked in mixing studies as source of mixing. Athough a mixing jet is not effective in a laminar mixing regime, approximately 70% of all flows occurring in nature are turbulent, including most processing in the chemical industry where, I believe FJFA can be used successfully.

In this presentation, I intend to discuss the general concept of the FJFA mixer sizing method, based on force (thrust) input from the propeller to the media.

The thrust is the result of force added to the system by the propeller [1,10]. The equation of motion is obtained from Newton's second law in the form which states that the sum of the external forces acting on the control volume of fluid equals the time rate of change of its momentum. (The equations used herein follow the absolute,mass-length-time, system)

$$F = m\,\Delta v \,/\, \Delta t$$

where: $m \,/\, \Delta t = \rho\, Q$.

The force exerted by the fluid in the X direction can be expressed:

$$F = \rho\, Q\, (v_{in} - v_{out}),$$

where: v_{in} = initial velocity,
v_{out} = final velocity.

If the thrust F is in the result of the action of the axial propeller on a stationary fluid volume (then $v_{out} = 0$), thrust can be expressed by the relationship:

$$F \propto \rho\, Q_j\, v_j,$$

where: ρ = density,
Q_j = primary flow from propeller,
v_j = jet velocity measured in the propeller blades plane.

The primary flow from the propeller is proportional to $\cancel{D^2}^{3}$ [13]:

D^3

$$Q_j \propto ND^3$$

and the jet velocity is proportional to speed N and diameter D:

$$v_j \propto N\,D$$

From the affinity laws the thrust force F and power P are given as:

$$F \propto \rho\, N^2 D^4, \qquad (1)$$

and

$$P \propto \rho\, N^3 D^5\,. \qquad (2)$$

Proportionality coefficients can be measured for each specific propeller. These coefficients are known as Power Number N_p and Thrust Number N_f.

Both the Thrust Number and Power Number are related to the Reynolds Number R_e. Although the N_p relation to R_e appears to be very well known, the N_f relation to R_e is relatively unknown. Recent studies have shown that in the turbulent zone there is very little variation of N_f with increasing R_e. In the transition zone, N_f decreases with decreasing R_e, and N_f continues an ever steeper declines with decreasing R_e in the laminar zone. Thus N_f has been found to behave similar to Flow Number N_q when the flow regime is changing from laminar to turbulent.

When Equations (1) and (2) are combined in terms of power (including proportionality coefficients N_p and N_f) they equal:

$$P = N_p / N_f\, F\, N\, D \qquad (3)$$

The force F is transmitted as a change in the velocity of the liquid mass at the tank walls and the bottom, creating pressure or so called wall shear stress τ_o [12]. An average value of the wall shear stress τ_o in the mixing tank can be expressed as equals:

$$\tau_o = F / S, \qquad (4)$$

where: S is the wall and the bottom wetted area.

When Equations (3) and (4) are combined, then specific energy ε (which equals $P/\rho V$) can be expressed as follows:

$$\varepsilon = P / \rho\, V = N_p / N_f (\tau_o/\rho)\, (S/V)\, N\, D. \qquad (5)$$

In the above equation the expression (S/V) is the inverse of the parameter known as Hydraulic Radius (R). When (S/V) in the Equation (5) is replaced by 1/R then ε equals:

$$\varepsilon = N_p / N_f (\tau_o /\rho)\, (N\, D / R) \qquad (6)$$

That R is a parameter which is related to the geometry of the tank is very well known in fluid mechanics. The larger the hydraulic radius R the more efficient is the tank geometry. Thus for the same volume of the tank the smaller wall area S will produce less resistance to flow. Therefore less energy would be needed (Equation 6) to reach the same process results. Another way of stating this is that tank geometry should be designed with the largest hydraulic radius, R, to produce the maximum flow for a given power input level.

Summarizing Equation 6, the Power Number N_p and Thrust Number N_f are functions of the type of mixer; ND relates to the speed of the mixer blade tips, and is also related to the fluid shear rate; Hydraulic Radius R describes tank geometry; the Shear Stress τ_o is the process design parameter which is a function of the process requirements and fluid properties.

DETERMINING FORCES FOR MIXING OF LOW VISCOSITY LIQUIDS.

When there are differences in composition (as density or viscosity) or temperature which create problems, mixing can be used as a mean of improving liquid uniformity in the tank/basin. The physical process which is required to achieve uniformity is called blending. The intensity of blending may be expressed by flow velocity, and simply related to scale of agitation, ranging from 1 to 10 [16]. Note that according to Dickey [3] fluid velocity in a tank is an essential parameter for quantifying mixer performance. When designing a mixing system to achieve the desired liquid uniformity, one needs to know what intensity of agitation is required, how many forces and how much power should be applied into subject tank/basin.

For determining the wall shear stress τ_o in the term of required velocity v for blending the very common equation from fluid dynamics can be used [2,18]:

$$\tau_O = \rho R (h/l) \qquad (7)$$

where l is the length of the water path in the tank, and R is the hydraulic radius.

To keep a liquid in the tank in motion an agitator creates a head, which equals the losses h in the tank. These losses h can be expressed as a function of the square of the hydraulic velocity v and a loss factor K:

$$h = K (v^2/2g) \qquad (8)$$

where the loss factor K is composed of the sum of bend losses K_b , friction losses K_f , contraction losses K_c , expansion losses K_{ex} etc. In general the value of the loss factor ($K = K_b + K_f$) in mixing tanks and basins is between 1 and 2.5, depending on their geometry.

The required level of v, which is an average hydraulic velocity, will depend on the needed intensity of agitation as derived from the 10 degrees scale identified for the different processes.

From Equations (7) and (8) we derive:

$$\tau_o = K \rho R/l (v^2/2g). \qquad (9)$$

Now, given the velocity level defined for process [16], we can use this relation for determining shear stress τ_o in the circulation channels, conduits, tanks and basins for Newtonian fluids.

Then the thrust which is required for blending can be determined from Equation (4) as follows:

$$F = \tau_o S,$$

and the specific energy ε can be defined from Equation (6).

THRUST NEEDED FOR MIXING FLUIDS POSSESSING YIELD STRESS.

The caverns formed around rotating impellers in the vessels containing yield stress fluids have been investigated by many authors [4,8]. One of the models was based on balancing the fluid centrifugal forces generated by the impeller ($\rho N^2 D^4$) with the retarding shear forces on the cavern's surface ($D_c^2 \tau_y$). Authors proposed a model for cavern size indicating that:

$$(D_c/D)^2 \propto (\rho N^2 D^2)/\tau_y \qquad (10)$$

where D_c is cavern size,

τ_y the fluid yield stress.

Etchells [5] called the dimensionless group ($\rho N^2 D^2/\tau_y$) a yield stress Reynolds number R_{ey}. The whole of the fluid in the vessel remains in motion when the cavern size reaches the vessel walls. It means that yield stress τ_y also reaches the walls.

When it is assumed that D_c^2 is proportional to the wetted surface of the mixing tank:

$$D_c^2 \propto S,$$

and existing yield stress τ_y acting on the walls equals τ_o, then Equation (10) is the same as a combination of Equations (1) and (4). This means that thrust needed to maintain the cavern volume, which approximately equals the size of the tank, is as follows:

$$F = \tau_y S \propto \rho N^2 D^4. \qquad (11)$$

An effective value for τ_y can be determined from the fluid rheology. This simple approach to define necessary thrust in the mixing tank for the fluids possessing a yield stress is applicable only for a turbulent flow in the vicinity of the propeller, where the propeller Reynolds number is 10^4, or more.

THE SUSPENSION OF SOLIDS PARTICLES

It was tested [7] for the low concentration of solids (C_w less than 10% by weight) that shear stress needed for just suspension particles off the bottom is:

$$\tau_o \propto C_w^{1/2} (\rho_s - \rho) d_p \qquad (12)$$

where: C_w is the solids concentration by weight expressed in percent,

ρ_s and ρ are the density of the solids and water in kg/ m^3,

d_p is diameter of a particle in m.

To define proper value for the shear stress needed for a particular process requires not only experience, but also evaluation of past studies and in many cases experimental efforts. Although defining proper shear stress value is difficult, it is the most important part of the mixer selection effort. The thrust based method produces very good results in establishing mixer applications for a given process, and when scale-up procedures from model tests are required.

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